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MEASURING EFFICIENCY AND TRADEOFFS IN ATTAINMENT OF EEO GOALS.(U)

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OF EEO GOALS**

by

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February 1982

*Office of the Assistant Secretary of the Navy Manpower and Reserve Affairs--
ODSM (CCP/EEO)

**The University of Pennsylvania

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MEASURING EFFICIENCY AND TRADEOFFS

IN ATTAINMENT OF EEO GOALS¹

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1. INTRODUCTION

This paper sketches a framework for evaluating the efficiency with which various units of the U.S. Navy attain their EEO goals. We focus on the conversion of "effort" or "input resources" into "outcomes" or "program outputs" reflecting the improvement in the representation of women and minorities in the various Navy units. To determine the efficiency of this conversion process we assume that, where needed, an EEO program will set out to provide the maximum amount of improvement in representation for a given level

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of resources; any such program is designated as being technically efficient. However, in the absence of known formulae of program operation, it is difficult to ascertain the level of outcome that should be expected and then determine efficiency by comparing attained outcomes with expected outcomes. In this paper, we shall propose to compare resource utilization and outcomes achieved by EEO programs among all units, and then evaluate the outcome levels that any program has achieved relative to what has been shown by other programs to be practically attainable with, at most, as many resources.

There are two useful by-products to the proposed exercise in measurement of EEO program efficiency. For each program being evaluated, a representative subset of efficient units is selected for comparison. By identifying this set of representative efficient units, we are able to provide guidance for further evaluation of the inefficient programs in order to help deduce what organizational action might help improve the efficiency of these programs.

A second useful result of the analysis is a set of tradeoff indicators reflecting the rate of substitution among program outcomes and input resources. Tradeoffs among outputs reflect possible substitution between the representation of different minority population groups because of limited opportunities within the organization, or limited supply of personnel in the relevant labor market. Tradeoffs among resources provide for the possibility of effecting substitutions between more expensive resources and less expensive resources, thereby achieving cost reduction.

The proposed approach incorporates a multiplicity of resource inputs and outcome indicators that need not be dimensionally commensurate. For example, inputs may be measured by the number of EEO staff, cost of program operation, number of opportunities created through internal or external recruitment, time of training required, etc. As indicators of program outcomes we shall usually choose, but not be restricted to, changes in the representation of women and minorities. Considerations of parity with the relevant external labor markets, the attainment of overall manpower staffing objectives, and the quality of personnel recruited will also enter into the analysis.

The proposed framework is intended to serve as part of the Navy's presently operating EEO monitoring system (see Niehaus and Nitterhouse [12]). Since 1975, comprehensive research into setting and monitoring the attainment of EEO goals has been underway

at the U.S. Navy.² The current Department of the Navy EEO Goals Accountability System (DONEAS) develops EEO goals based on estimated relevant labor market supply ratios and organizational data (see Atwater, Niehaus, and Sheridan [1] and [2]). These EEO goals are developed separately for each organizational unit. By way of contrast, the present paper is intended to provide a means to effect comparisons among organizations engaged in evaluation in order to measure the efficiency of their performance in goal attainment.

The organization of the paper is as follows. In section 2 we review the formulation of the relative efficiency problem, drawing primarily on the work by Charnes, Cooper and co-workers [5,7]. In section 3 we provide a numerical illustration of the approach. In section 4 we introduce a criteria for partitioning the set of EEO programs into comparison groups for further evaluation and case studies, and conclude in section 5 by suggesting application of the method for linking analyses in internal and external labor markets.

While some portions of this paper are methodological, it should be noted that our purpose here is not to provide the general formulation of the relative efficiency problem,³ rather, the objective is to outline the basic framework of analysis as a basis of experimental application within the Navy.

2. MEASUREMENT OF RELATIVE EFFICIENCY

For any EEO program (unit) being evaluated, a corresponding practically attainable point on the frontier of EEO program outcomes consists of the maximum output (set of outcomes, or combinations thereof) that has been shown by all units to be practically attainable with, at most, as many EEO input resources (set of inputs, or combinations thereof). Consider a partitioning of the Navy into n administrative units with associated EEO programs, each with m input resources and s program outcome indicators. Let x_{ij} be the amount of input i to EEO program j , and y_{rj} a measure of program outcome r (output) of EEO program j .

²For a summary of the initial phases of this research see Charnes, Cooper, Lewis, and Niehaus [6]. Also see Chapters III and IV of Niehaus [11].

³See, for example, Banker, Charnes, Cooper and Schinnar [3] for a nonlinear formulation and access to measuring returns to scale, and Schinnar [15] for a constrained formulation of the problem.

Following the Charnes, Cooper et al [5,7] non-Archimedean characterization of the efficiency problem,⁴ we can describe the EEO program frontier by a piecewise linear envelope constructed from solutions to the following linear programming problem

$$\begin{aligned}
 &\text{maximize} && z_0 + \epsilon \sum_{r=1}^s \delta_r^- + \epsilon \sum_{j=1}^m \delta_j^+ \\
 &\text{subject to} && - \sum_{j=1}^n y_{rj} \lambda_j + y_{r0} z_0 + \delta_r^- = 0, \quad \text{all } r \quad (1) \\
 &&& \sum_{j=1}^n x_{ij} \lambda_j + \delta_j^+ = x_{i0}, \quad \text{all } i \quad (2) \\
 &&& \lambda_j, \delta_r^-, \delta_j^+ \geq 0, \quad z_0 \text{ unrestricted}
 \end{aligned}$$

where $r = 1, \dots, s$; $i = 1, \dots, m$; the symbol $\epsilon > 0$ and less than every positive number in the base field, is the infinitesimal used to generate the non-Archimedean ordered extension field (see [4] pp. 756-757) and "o" indicates the subscripts of one of the $j=1, \dots, n$ units that are being evaluated. The constraint set (1) envelops the outputs from above, while the constraint set (2) envelops the inputs from below. The scalar z_0 provides the proportionate factor increase in outputs (y_{i0}) that has been shown by other EEO programs to be practically attainable with at most (x_{i0}) input resources,⁵ while efficiency is, in turn, measured by the reciprocal of z_0^* . By the non-Archimedean efficiency theorem [5], an EEO program is efficient if and only if

$$z_0^* + \epsilon \sum_{r=1}^s \delta_r^{*-} + \epsilon \sum_{j=1}^m \delta_j^{*+} = 1, \text{ which implies, in turn, that}$$

$$\delta_r^{*-} = \delta_j^{*+} = 0, \text{ all } r \text{ and } j, \text{ and therefore that } z_0^* = 1.$$

The set of weights (λ_j) defines the piecewise linear frontier constructed from facets of a polytope whose extreme points are efficient decision-making units. For each unit, the corresponding basic (λ_j^*) identify the representative group of efficient units on the frontier. Several different units may all be in the same cone generated from the same set of (λ_j^*). In section 4 we shall

⁴This way of proceeding also clears up an ambiguity noticed by Färe and Lowell [9] in the pioneering work of Farrell [10].

⁵Assuming complete managerial discretionary control of input resources. See [5] and [14] for a discussion of non-discretionary resources.

refer to such sets of units as "comparison groups" of EEO programs, and describe some of their properties.

The non-Archimedean ratio form⁶ of the efficiency problem is especially amenable to a "cost/benefit" interpretation of the efficiency score and is obtainable from the dual program of (1)-(2).

$$\text{minimize} \quad g_0 = \sum_{i=1}^m w_i x_{i0} \quad (3)$$

$$\text{subject to: } - \sum_{r=1}^s u_r y_{rj} + \sum_{i=1}^m w_i x_{ij} \geq 0 \quad j = 1, \dots, n \quad (4)$$

$$\sum_{r=1}^s u_r y_{r0} = 1 \quad (5)$$

$$u_r, w_i \geq \epsilon,$$

which may be transformed to the ratio problem via a change of variables $u_r = t\mu_r$, $w_i = t\omega_i$, $t > 0$

$$\text{minimize} \quad f_0 = \left(\sum_{i=1}^m \omega_i x_{i0} \right) / \left(\sum_{r=1}^s \mu_r y_{r0} \right) \quad (6)$$

$$\text{subject to: } \left(\sum_{i=1}^m \omega_i x_{ij} \right) / \left(\sum_{r=1}^s \mu_r y_{rj} \right) \geq 1, \text{ all } j \quad (7)$$

$$\mu_r, \omega_i \geq \epsilon \sum_{r=1}^s \mu_r y_{r0} \quad \text{all } r, i$$

The measure of efficiency in (6), reflects the minimum of a ratio of "weighted" inputs to "weighted" outputs subject to the condition that similar ratios for every unit be at least one. These weights here are not preassigned, however; instead, they are obtained as solutions to the above optimization problem in view of the data on observed EEO program operations. Units incurring least amount of input, a so-called virtual input, per unit of (virtual) output are relatively efficient. All other units are inefficient.

⁶In [5] and [7] the ratio form (6)-(7) constitutes the initial formulation from which the envelopment form (1)-(2) is derived. We have chosen here to motivate the problem by the envelopment procedure and show (6)-(7) and its associated interpretations as a derivation instead.

The dual variables u_r and w_i provide access to tradeoff interpretations. For every efficient unit found in the basis of an optimal solution of (1)-(2), the corresponding constraint set in (4) is tight.

$$-\sum_{r=1}^s u_r^* y_{rj} + \sum_{i=1}^m w_i^* x_{ij} = 0 \quad (8)$$

Thus, for a given level of efficiency, defined by $1/z_0^*$, the ratio $-w_i^*/w_k^*$ gives the trade-off rate between inputs k and i on the corresponding facet of the frontier (holding all other inputs and outputs constant), while the ratio $-u_r^*/u_q^*$ gives the tradeoff rate between outputs q and r (holding all other inputs and outputs constant).⁷ Tradeoffs in the input space (2) could reflect a substitution between, e.g., EEO program staffing levels and the length of a training period for minority recruits. Tradeoffs in the output space (1) could reflect a substitution between, e.g., the attainment of representation goals for Black and Hispanic populations, respectively.

3. EXAMPLE

A data base on 18 of the Navy's organizational units that meet a minimum size requirement of 1000 civilian employees is used to illustrate the method of analysis. We focus in this example on the representation of two minority groups, Blacks and Hispanics, in the job category of managers and administrators, grade 9-12, as indicators of EEO program outputs, and on the level of EEO staffing and the recruitment opportunity rates as input data.

Table 1 contains the input and output measures corresponding to each unit.

Input Data: The input data are designed to reflect, on the one hand, the level of EEO program resources available and, on the other hand, the availability of opportunities for minority personnel. The two inputs are: EEO STAFF = The number of EEO specialists (or full-time equivalent) per 1000 employees for each organizational unit.

OPP RATE = The general⁸ rate of recruitment (promotion and hiring) into the managers and administrators grades (GS 9 through 12).

⁷For a related discussion of tradeoffs see [8]. Bear in mind that this does not apply to efficient corner units which may be part of several facets.

⁸Recruitment of minority and non-minority personnel.

Table 1. Input and Output Data for Eighteen Organizational Units in the U.S. Navy

| I.D. Number | Outputs | | Inputs | |
|----------------|-------------|----------------|---------------|-------------|
| | Blacks R | Hispanics R | EEO STAFF* | OPP RATE |
| 1 | 1.098 | 0.976 | 0.985 | 0.104 |
| 2 | 0.997 | 0.884 | 1.897 | 0.110 |
| 3 | 1.032 | 1.138 | 0.672 | 0.125 |
| 4 | 1.133 | 1.070 | 1.589 | 0.115 |
| 5 | 1.146 | 0.862 | 0.827 | 0.109 |
| 6 | 1.651 | 0.978 | 1.875 | 0.108 |
| 7 | 1.120 | 1.035 | 1.905 | 0.108 |
| 8 | 0.993 | 1.227 | 1.769 | 0.108 |
| 9 | 1.104 | 0.968 | 0.676 | 0.121 |
| 10 | 1.114 | 0.634 | 0.000 | 0.140 |
| 11 | 1.186 | 1.025 | 1.718 | 0.158 |
| 12 | 1.062 | 1.000 | 0.472 | 0.135 |
| 13 | 0.868 | 1.093 | 0.656 | 0.126 |
| 14 | 0.971 | 1.040 | 2.386 | 0.128 |
| 15 | 0.667 | 0.967 | 0.531 | 0.101 |
| 16 | 1.262 | 0.443 | 1.292 | 0.175 |
| 17 | 0.914 | 1.064 | 1.261 | 0.136 |
| 18 | 1.362 | 1.000 | 0.682 | 0.145 |

* All quantities are stated in units of 1,000 employees

The figures reflect an average annual rate based on FY72-FY78 data. These are shown in the last two columns of table 1. Output Data: The output data are intended to reflect relative progress (or the lack of progress) in attainment of minority representation goals between FY78 and FY79. In table 2 we show the percentage of EEO⁹ goals attained for Black and Hispanic personnel by each unit in FY78 and FY79, i.e.,

⁹These goals are based on undifferentiated Civilian Labor Force (CLF) ratios required for reporting by the Equal Employment Opportunity Commission (EEOC) rather than the more appropriate Relevant Labor Force ratios that consider occupational and wage availability. Both RLF and CLF standards are incorporated in the Navy's DONEAS described in [12]. The first operational version of Navy RLF data is provided in [2].

Table 2. Percentage of EEO Goals Attained by Navy Units for FY78-FY79

| I.D. Number | Blacks | | Hispanics | |
|----------------|--------|--------|-----------|--------|
| | FY78 | FY79 | FY78 | FY79 |
| 1 | 44.700 | 49.090 | 52.240 | 51.000 |
| 2 | 98.040 | 97.760 | 39.420 | 34.840 |
| 3 | 48.480 | 50.030 | 36.460 | 41.500 |
| 4 | 39.230 | 44.440 | 38.320 | 41.020 |
| 5 | 36.440 | 41.750 | 48.520 | 41.820 |
| 6 | 22.430 | 37.070 | 101.200 | 98.940 |
| 7 | 26.440 | 29.600 | 36.280 | 37.550 |
| 8 | 83.780 | 83.190 | 35.790 | 43.920 |
| 9 | 40.400 | 44.590 | 20.260 | 19.610 |
| 10 | 79.130 | 88.120 | 30.970 | 19.650 |
| 11 | 53.200 | 63.120 | 15.500 | 15.880 |
| 12 | 36.040 | 38.260 | 0 | 0 |
| 13 | 21.750 | 18.880 | 14.380 | 15.720 |
| 14 | 68.570 | 66.610 | 54.400 | 56.550 |
| 15 | 62.730 | 41.830 | 35.690 | 34.500 |
| 16 | 50.220 | 63.390 | 28.450 | 12.610 |
| 17 | 50.500 | 46.180 | 46.320 | 49.270 |
| 18 | 37.900 | 51.620 | 0 | 0 |

*Based on undifferentiated Civilian Labor Force (CLF) hiring goals consistent with EEOC MD-702. (See Footnote 8)

$$\frac{A_{79}}{G_{79}} = \frac{\left(\begin{array}{l} \text{Actual \% representation of} \\ \text{a minority group for FY79} \end{array} \right)}{\left(\begin{array}{l} \text{Goal for \% representation of} \\ \text{a minority group set for FY79} \end{array} \right)}$$

The output indicator

$$R = \frac{A_{79}/G_{79}}{A_{78}/G_{78}}$$

then reflects progress in goal attainment over the 78-79 transition period. $R > 1$ indicates progress, $R = 1$ reflects no change, and $R < 1$ shows an increasing divergence between the EEO goal and the

Table 3. Efficiency Measures and Slack Variables

| ID Number | Output Factor Increase (z_o) | Efficiency Rating ($1/z_o$) | Slack Variables | | | |
|-----------|----------------------------------|-------------------------------|-----------------|---------------|-----------|------------------|
| | | | Black Rep. | Hispanic Rep. | EEO Staff | Opportunity Rate |
| 1 | 1.000 | 1.000 | 0 | 0 | 0 | 0 |
| 2 | 1.294 | 0.773 | 0 | 0 | 0.050 | 0 |
| 3 | 1.000 | 1.000 | 0 | 0 | 0 | 0 |
| 4 | 1.094 | 0.914 | 0 | 0 | 0 | 0 |
| 5 | 1.000 | 1.000 | 0 | 0 | 0 | 0 |
| 6 | 1.000 | 1.000 | 0 | 0 | 0 | 0 |
| 7 | 1.099 | 0.910 | 0 | 0 | 0.098 | 0 |
| 8 | 1.000 | 1.000 | 0 | 0 | 0 | 0 |
| 9 | 1.002 | 0.998 | 0 | 0 | 0 | 0 |
| 10 | 1.000 | 1.000 | 0 | 0 | 0 | 0 |
| 11 | 1.464 | 0.683 | 0 | 0 | 0 | 0 |
| 12 | 1.020 | 0.981 | 0 | 0 | 0 | 0 |
| 13 | 1.068 | 0.936 | 0 | 0 | 0 | 0 |
| 14 | 1.350 | 0.741 | 0 | 0 | 0.268 | 0 |
| 15 | 1.000 | 1.000 | 0 | 0 | 0 | 0 |
| 16 | 1.535 | 0.651 | 0.449 | 0 | 0 | 0 |
| 17 | 1.269 | 0.788 | 0 | 0 | 0 | 0 |
| 18 | 1.060 | 0.944 | 0 | 0 | 0 | 0 |

actual representation of a minority group. The output indicators (R) for Blacks and Hispanics are shown in columns 2 and 3 of table 1. Observe that only six of the EEO programs have registered progress for both Blacks and Hispanics; in two cases there was a decrease in the representation of both groups, and in ten units progress in one group was accompanied by diminished representation of the other group.¹⁰

¹⁰ As David Sherman has shown in his analysis of the relative efficiency of health service organizations, ratios should be used with caution. Our analysis here is illustrative however, and in actual application nonratio output quantities could be readily incorporated because of the linear programming methodology.

Table 3 gives a summary of the results obtained from an application of program (1)-(2) to the data in table 1. For each EEO program, identified by an ID number in the first column, we show the proportional factor increase in output (z_0^* , in column 2), the input efficiency score ($1/z_0^*$, in column 3), and the slack variables associated with the outputs (column 4 and 5) and the inputs (columns 6 and 7). By scanning column 3 we note that there are seven efficient units (#1, 3, 5, 6, 8, 10, 15), all with $z_0^* = 1$ and zero slacks. Units #2, 11, 14, 16 and 17 have efficiency scores of less than 0.8, with the remaining six units nearly efficient (i.e., $0.9 \leq z_0^* < 1$). Consider for example unit 14 which posted an efficient score of 0.741. The associated $z_0^* = 1.35$ suggests that, in this case, the evidence from the operations by others indicates that a 35% increase in the representation of all minorities should have been attained with no more input resources. In fact, the slack of .268 associated with EEO staff implies that, in addition, this 35% increase in representation was attainable with a concomitant reduction of .268 from the present level of 2.386 EEO staff per 1000 employees.

Observe that in the definition of the output indicator R, three of the four terms (G_{78} , A_{78} , G_{79}) used to compute R can be regarded as fixed. It follows then that z_0 constitutes the proportional factor increase in A_{79} , i.e., in the actual representation during the FY79 evaluation year. Therefore, by subtracting unity from the entries in the second column of table 3, we can obtain the growth rate in minority representation that has not been achieved but has been shown attainable by other units.

There is also a difference in the number of civilian employees among the Navy units. The first nine units have more than 10,000 civilian employees each, while the size of the civilian labor-force of the remaining nine units range from 1000 to 10,000 employees. The nine larger units are on the average more efficient (.96) than the nine smaller units (.86). This suggests the presence of some economics of scale in EEO programs. In order to obtain direct measures of return to scale, a recourse to the bi-extremal variant of the method developed by Banker, Charnes, Cooper and Schinnar [3] is required. However, this would necessitate a parametric characterization of the "EEO program production function," which is not required by the present analysis.

In closing, we should like to underscore the importance of proper selection of variables for the interpretation of results. By scanning the rows of table 2 we note that in all but one efficient unit, progress has been made in approaching the EEO goals of at least one minority group. Unit 15 has registered a decline for both minority groups, but because it has employed few inputs it is found efficient. Observe, however, that limited amounts of input can have, in this case, two interpretations: (i) a low

opportunity rate implies few opportunities for internal and external recruitment irrespective of the level of EEO effort; and (ii) a small EEO staff may not be able to provide the necessary assistance to maintain, let alone increase, minority representation. This may explain, at least partly,¹¹ the lack of progress in goal attainment achieved by unit 15. Nonetheless, in the context of the present analysis, "efficiency" suggests that, given the limited opportunities for recruitment and the small EEO staff, the loss in minority representation might have been even greater had the EEO staff not used its resources efficiently.

4. COMPARISON GROUPS

The second objective of this paper is to provide guidance for further evaluation of EEO programs in order to identify, e.g., organizational impediments that may be the source of inefficiency in the EEO programs. Such studies will provide the necessary information for effecting actual improvements in the operation of EEO programs. In this section we introduce a criteria for choosing a comparison group in order to evaluate a given unit.

We define a comparison group as a set of units which share the same representative set of efficient units on the frontier; the representative set may include "dummy units" associated with slack variables. As will be shown below, the dual variables associated with any member of such group are scalar multiples of the dual variables of any other member of the group. Consequently, it is assumed that for designated contours of efficiency levels, the rates of tradeoff among inputs and outputs are the same for all members of a comparison group. This suggests that, in addition to sharing the same subset of representative efficient units on the frontier,¹² the comparison group also shares a "technology" for converting resource inputs into EEO program outcomes.

We focus now on identifying the members of the comparison group for a unit whose inputs and outputs are given by $\{x_{10}\}$ and $\{y_{r0}\}$. We assume that a solution of program (1)-(2) has been obtained so that z_0^* , $\{\lambda_j^*\}$, the slacks, $\{u_r^*\}$ and $\{w_i^*\}$ are readily available. In addition, we retain the "inverse of the optimal basis" in (1)-(2) and denote by N the matrix containing its first $m+s-1$ rows.

¹¹Alternatively, the initial set of goals may have been too ambitious.

¹²Note, however, that the degree to which a particular efficient unit is representative of an inefficient unit will vary across members of the group.

Proposition 1: A unit k is a member of a comparison group defined with respect to unit o if, and only if,

$$\sum_{r=1}^s u_r^* y_{rk} > 0 \quad (9)$$

$$z_k \sum_{r=1}^s N_{jr} y_{rk} - \sum_{i=s+1}^{s+m} N_{ji} x_{ik} \geq 0 \quad (10)$$

for all j (correspond to basic λ_j and slacks), where

$$z_k = \frac{\sum_{i=1}^m w_{io}^* x_{ik}}{\sum_{r=1}^s u_r^* y_{rk}} \quad (11)$$

Proposition 2: If (9) and (10) hold, then (11) gives z_k^* ; the left-hand side of inequality (1) gives the value for λ_j^* and the slacks; and

$$w_{ik}^* = w_{io}^* / \left(\sum_{r=1}^s u_r^* y_{rk} \right) \quad (12)$$

$$u_{rk}^* = u_{ro}^* / \left(\sum_{r=1}^n u_r^* y_{rk} \right) \quad (13)$$

The proofs for the above propositions follow from the derivation of an algorithm in Schinnar [14].¹³ Equation (9) is derived from the optimality condition for the linear programming problem (1)-(2), in which the y_{ro} and the x_{io} have been replaced by y_{rk} and x_{ik} , respectively. It is also a sufficient condition for constructing a basic solution for $\{y_{rk}, x_{ik}\}$ from the optimal basis associated with an optimal solution of (1)-(2) for $\{y_{ro}, x_{io}\}$. Equation (10) follows from the feasibility test for the new basis. Also note that z_k in (11) coincides with f_k in (6), and that within a comparison group the dual evaluators are unique up to multiplication by a scalar; cf. equations (12)-(13).

The above simple tests provide a way by which to identify the entire comparison group for a given EEO program from a single linear programming solution. This comparison group will include

¹³These results are also suggested in an earlier draft of Charnes, Cooper and Rhodes [7].

both efficient and inefficient units. Several options are thus available for proceeding with case-studies to identify the determinants of inefficiency:

1. A comparison of inefficient EEO programs with other inefficient programs in the comparison group in order to observe common features that may help explain the observed inefficiency.
2. A comparison of efficient EEO programs among themselves in order to identify common factors that may help explain their efficiency.
3. A comparison of the inefficient EEO programs with the efficient subset of units in order to observe systematic differences between efficient and inefficient units.

We now continue with the illustrative example of section 3. Table 4 identifies the membership in six comparison groups into which the set of 18 EEO programs, evaluated in table 3, may be divided. Each row in the table marks the inefficient and efficient members of a group. Note that the inefficient units are members in one group only, while the efficient units appear in several comparison groups when they provide the corners of the facets forming the efficiency frontier. For example, in group D, two inefficient units are associated with three efficient units that are their representatives on the efficiency frontier. Thus, a case study designed to assess the sources of inefficiency in units 12 and 13 should include comparisons of these units with units 3, 10, and 15. Further specificity in the comparisons can be obtained by reference to the weights associated with each of the efficient units; the weights are the primal variables obtained from program (1)-(2) and are displayed in table 5. Efficient units with larger weight enter more "heavily" into the evaluation of an inefficient unit and are therefore more suitable for effecting comparisons between inefficient and efficient program units. Thus, the inefficient unit 12 should be primarily compared with the efficient unit 3, and the inefficient unit 13 should be primarily compared with the efficient unit 15.

Note also that groups A and E have only two efficient units each, whereas the other groups include three. This results from the presence of a (positive) slack variable in the basis of groups A and E and implies, in turn, that their associated facet is not efficient. However, this need not preclude comparisons between the inefficient and efficient units in these groups.

A final observation is made with reference to the tradeoff between Black and Hispanic representation at the efficiency frontier of each comparison. The last column of the table shows the substitution possibilities between the two minority groups while remaining on the efficiency frontier. These figures are obtained from the ratios of the dual variables associated with

Table 4. Membership in Six Comparison Groups of Navy Units and Tradeoff Between Black and Hispanic Representation

| Group | Inefficient Units | | | | | | | | | | Efficient units | | | | | | | Tradeoffs between Blacks and Hispanics | |
|-------|-------------------|---|---|---|----|----|----|----|----|----|-----------------|---|---|---|---|---|----|--|------|
| | 2 | 4 | 7 | 9 | 11 | 12 | 13 | 14 | 16 | 17 | 18 | 1 | 3 | 5 | 6 | 8 | 10 | | 15 |
| A | X | | X | | | | | X | | | | | | | X | X | | | -2.6 |
| B | | X | | | X | | | | | | | X | | | X | X | | | -2.4 |
| C | | | | X | | | | | | | X | | X | X | | | X | | -0.7 |
| D | | | | | | X | X | | | | | | X | | | | X | X | -2.9 |
| E | | | | | | | | | X | | | | | | X | | X | | 0 |
| F | | | | | | | | | | X | | X | | | | X | | X | -3.4 |

output indicators in program (3)-(4). For example, in group A (units 2, 7, and 14), the tradeoff rate of 2.6 suggests that a loss of two Black employees should be replaced by a gain of approximately five Hispanics if the EEO program is being operated efficiently. This indicates that the EEO programs in comparison group A are more sensitive to progress in the representation of Blacks and Hispanics, and may suggest a greater supply of Hispanics in the relevant external labor market of qualified personnel. In comparison group D, the situation is reversed: fewer Hispanics are required to replace Blacks in order to maintain the efficiency rating of the EEO programs. These tradeoff rates reflect the slopes of the facets of the piece-wise linear frontier and are therefore meaningful in the context of a discussion of comparison groups. Any attempt to attribute these tradeoff rates to specific efficient units is erroneous because of their membership in several groups. (See Schinnar [13] for further discussion.)

5. CONCLUSION

We have presented a framework for monitoring the relative efficiency of EEO programs in the Navy, and demonstrated how the information can be used to guide further evaluation of the "cost effectiveness" of these programs in order to obtain improvements in EEO program operations. We have shown an example of an application of the method, using preliminary data on 18 organizational units in the Navy. The reader should bear in mind, though, that this application is intended for illustration only, and the figures should not be used to evaluate any of the units involved. For this purpose, a more complete data base is required, coupled with a better specification of the input and output indicators. For

Table 5. Representation Weights for Efficient Units

| Units Evaluated | Efficient Units | | | | | | |
|--------------------|-----------------|-------|-------|-------|-------|-------|-------|
| | #1 | #3 | #5 | #6 | #8 | #10 | #15 |
| 1 | 1.000 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0.425 | 0.594 | 0 | 0 |
| 3 | 0 | 1.000 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0.437 | 0 | 0 | 0.183 | 0.461 | 0 | 0 |
| 5 | 0 | 0 | 1.000 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 1.000 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0.360 | 0.640 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 1.000 | 0 | 0 |
| 9 | 0 | 0.444 | 0.456 | 0 | 0 | 0.112 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 1.000 | 0 |
| 11 | 1.235 | 0 | 0 | 0.166 | 0.108 | 0 | 0 |
| 12 | 0 | 0.617 | 0 | 0 | 0 | 0.335 | 0.108 |
| 13 | 0 | 0.445 | 0 | 0 | 0 | 0.018 | 0.673 |
| 14 | 0 | 0 | 0 | 0.205 | 0.981 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 1.000 |
| 16 | 0 | 0 | 0 | 0.689 | 0 | 0.718 | 0 |
| 17 | 0.438 | 0 | 0 | 0 | 0.295 | 0 | 0.580 |
| 18 | 0 | 0.038 | 1.036 | 0 | 0 | 0.196 | 0 |

example, in subsequent applications, the output measures that relate to EEO goals will be computed using the more appropriate data which is based on relevant labor force supply statistics.

Further conceptual developments of the model would involve a more explicit linkage between the organizational structure of opportunities and the availability of qualified personnel in the relevant labor market, as well as the incorporation of data (such as the size of EEO programs) that is not only of an "input" or "output" variety.

REFERENCES

1. Atwater, D.M., R.J. Niehaus and J.A. Sheridan, "EEO Goals Development in the Naval Sea Systems Command," OASN(M,RA&L) Research Report No. 35 (Washington, D.C.: Office of the Assistant Secretary of the Navy [Manpower, Reserve Affairs and Logistics], 1979).

2. Atwater, D.M., R.J. Niehaus and J.A. Sheridan, "EEO External Relevant Labor Force Analysis," OASN(M,RA&L) Research Report No. 37 (Washington, D.C.: Office of the Assistant Secretary of the Navy [Manpower, Reserve Affairs and Logistics], 1980).
3. Banker, R., A. Charnes, W.W. Cooper and A.P. Schinnar, "A Bi-Extremal Principle for Frontier Estimation and Efficiency Evaluation," Management Science, forthcoming.
4. Charnes, A. and W.W. Cooper, Management Models and Industrial Application of Linear Programming, (New York: Wiley, 1961).
5. Charnes, A., W.W. Cooper, A. Lewin, R. Morey and J. Rousseau, "Efficiency analysis with non-discretionary resources," Center for Cybernetic Studies Research Report 379, The University of Texas, Sept. 1980.
6. Charnes, A., W.W. Cooper, K.A. Lewis and R.J. Niehaus, "Design and Development of Equal Employment Opportunity Human Resources Planning Models," NPDRC TR79-141 (San Diego: Navy Personnel Research and Development Center, 1979).
7. Charnes, A., W.W. Cooper and E. Rhodes, "Measuring the Efficiency of Decision-Making Units," European Journal of Operational Research, Vol. 2, 1978, pp. 429-444.
8. Charnes, A., W.W. Cooper, A.P. Schinnar and N.E. Terleckyj, "A Goal Focusing Approach to Analysis of Tradeoffs Among Household Production Outputs," American Statistical Association 1979 Proceedings of the Social Statistics Section, 1979, pp. 174-199.
9. Färe, R. and C.A.K. Lovell, "Measuring the Technical Efficiency of Production," Journal of Economic Theory, 1978, pp. 150-162.
10. Farrell, M.J., "The Measurement of Productive Efficiency," J. Roy. Statist. Soc. Ser. A, Vol. III, 1957, pp. 253-390.
11. Niehaus, R.J., Computer-Assisted Human Resources Planning, (New York: Wiley-Interscience, 1979).
12. Niehaus, R.J. and D. Nitterhouse, "Planning and Accountability Systems for EEO and Affirmative Action Policy," OASN (M,RA&L) Research Report No. 38 (Washington, D.C.: Office of the Assistant Secretary of the Navy [Manpower, Reserve Affairs and Logistics], 1980).
13. Schinnar, A.P., "Measuring Productive Efficiency of Public Service Provision," CDSE/HUD Project Report, Fels Discussion Paper No. 143, University of Pennsylvania, July 1980.
14. Schinnar, A.P., "An Algorithm for Measuring Relative Efficiency," Discussion Paper No. 144, University of Pennsylvania, August 1980.
15. Schinnar, A.P., "Constrained Isoquant Analysis of Efficiency Frontiers," Fels Discussion Paper No. 145, University of Pennsylvania, August 1980.

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absence of known formulae of program operation, it is difficult to ascertain the level of outcome that should be expected and then determine efficiency by comparing attained outcomes with expected outcomes. In this paper, we shall propose to compare resource utilization and outcomes achieved by EEO programs among all units, and then evaluate the outcome levels that any program has achieved relative to what has been shown by other programs to be practically attainable with, at most, as many resources.

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